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Solutions to Cosmological Problems with Energy Conservation and Varying c, G and Λ

P. Gopakumar and G.V. Vijayagovindan
School of Pure and Applied Physics,
Mahatma Gandhi University,
Kottayam - 686 560,
India.

Abstract

The flatness and cosmological constant problems are solved with varying speed of light c, gravitational coupling strength G and cosmological parameter Λ, by explicitly assuming energy conservation of observed matter. The present solution to the flatness problem is the same as a previous solution in which energy conservation was absent.

As an alternative to the inflationary model of the universe, varying speed of light theories [1, 2, 3, 4, 5] had been introduced. Experimental observation of the variation of fine structure constant with time has been indicated by quasar absorption spectra[6]. Variations of fine structure constant could be interpreted as variation of the speed of light or of the fundamental charge, e. In the model of Moffat[1], variation of speed of light arises due to the spontaneous breakdown of local Lorentz invariance in the early universe. In a later model[2], he introduced a dynamical mechanism of varying speed of light (VSL) by working in a bi- metric theory. Kristsis[7, 8] has given a VSL theory in 3+1 dimensions by starting from a string theory motivated theory of branes. Albrecht and Magueijo[3] and Barrow[4] consider not only models with VSL but also those allowing G and Λ to vary with respect to time in the conservation equations.

In this paper we consider a VSL theory in which the energy-momentum of matter is conserved. We have reformulated the solutions to the cosmological problems on this basis.

1 Flatness Problem

Albrecht and Magueijo proposed that a time varying speed of light c should not introduce changes in the curvature terms in the Einstein's equations in the cosmological frame and that Einstein's equations must still hold. Assuming that matter behaves as a perfect fluid, the equations of state can be written as,

$$P = (\gamma - 1)\rho c^2(t). \tag{1}$$

Friedmann equations for a homogeneous space time, with c and G, as functions of time are,

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G(t)\rho - \frac{Kc^2(t)}{a^2} \tag{2}$$

$$\ddot{a} = -\frac{4}{3}\pi G(t)(\rho + \frac{3p}{c^2(t)})a \tag{3}$$

where ρ and p are density and pressure of the matter, and K is the metric curvature parameter. Combining Eq.(2) and Eq.(3), the generalized conservation equation is,

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + \frac{p}{c^2}) = -\rho\frac{\dot{G}}{G} + \frac{3Kc\dot{c}}{4\pi G a^2} \tag{4}$$

We assume the conservation of ordinary matter; ie., the left hand side of Eq.(4) is zero. Thus the variations in c and G are such that the right hand side of Eq.(4) is identically zero. Assuming $\rho \propto a^{-3\gamma}$ and $c(t) = c_0 a^n$,where c_0 and n are constants. The solution of the right hand side of Eq.(4), for $3\gamma + 2n - 2 \neq 0$ is

$$G = \frac{3K(c_0)^2 n}{4\pi\rho_0}\frac{a^{3\gamma+2n-2}}{3\gamma + 2n - 2} + B \tag{5}$$

and for, $3\gamma + 2n = 2$ is

$$G = \frac{3K(c_0)^2 n}{4\pi\rho_0} \ln a + B \tag{6}$$

where B is a constant of integration. Thus the Friedmann equation for the $3\gamma + 2n \neq 2$ case becomes

$$\frac{\dot{a}^2}{a^2} = B' a^{-3\gamma} + \frac{K(c_0)^2 a^{2n-2}(2 - 3\gamma)}{(3\gamma + 2n - 2)} \tag{7}$$

where B'is a constant. The curvature term in Eq.(2) vanishes as the scale factor evolves, if $\ddot{a} > 0$ ie., $\rho + \frac{3p}{c^2} < 0$. Also the Eq.(4) gives the solution,

$$\rho \propto a^{-3\gamma} \text{ for } \ddot{G} = \ddot{c} = 0 \text{ if } \rho + \tfrac{p}{c^2} \geq 0$$

Using the equation of state, Eq.(1), these conditions imply

$$0 \leq \gamma < \tfrac{2}{3}$$

The scale factor evolves as

$$a(t) \propto t^{\frac{2}{3}\gamma} \text{ if } \gamma > 0$$

and

$$a(t) \propto \exp\left(H_0(t)\right. \text{ if } \gamma = 0$$

For $\gamma < \frac{2}{3}$, the requirement $\rho + \frac{3p}{c^2} < 0$ implies $p < -\frac{1}{3}\rho c^2$, ie., the curvature term will vanish at large a only if the matter stress is gravitationally repulsive. From Eq.(7) it can be seen that the flatness problem can be solved for,

$$n \leq \frac{1}{2}(2 - 3\gamma). \tag{8}$$

This is exactly the same inequality, which Barrow derived without assuming energy conservation of ordinary matter[4].

2 Lambda Problem

To incorporate the cosmological constant term into the Friedmann equation, a vacuum stress is considered obeying the equation of state,

$$p_\Lambda = -\rho_\Lambda c^2, \tag{9}$$

with

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G} \geq 0. \tag{10}$$

The Friedmann equation containing $\rho_\Lambda$is,

$$\frac{\dot{a}^2}{a^2} = \frac{8}{3}\pi G(\rho + \rho_\Lambda) - \frac{Kc^2}{a^2}. \tag{11}$$

As the universe expands the term containing ρ_Λ should dominate. But observationally it is very small. This is the Λ problem. The conservation Eq.(4) generalised to include ρ_Λ is,

$$\dot{\rho} + 3\frac{\dot{a}}{a}[\rho + \frac{p}{c^2}] = -\dot{\rho_\Lambda} - (\rho + \rho_\Lambda)\frac{\dot{G}}{G} + \frac{3Kc\dot{c}}{4\pi G a^2}. \tag{12}$$

We assume a form for the variation of G and Λ in terms of the scale factor as, $G = G_0 a^q$ and $\Lambda = \Lambda_0 a^s$ where Λ_0,q, G_0 and s are constants. Now Eq.(12) can be written as,

$$\dot{\rho} + 3\frac{\dot{a}}{a}\frac{p}{c^2} + \frac{\dot{\Lambda}c^2}{8\pi G} = -(\rho + \rho_\Lambda)\frac{\dot{G}}{G} + \frac{3Kc\dot{c}}{4\pi G c^2} - \frac{2\Lambda c\dot{c}}{8\pi G} + \frac{\Lambda c^2 \dot{G}}{8\pi G^2} \tag{13}$$

Assuming conservation of ordinary matter, we put the left hand side and right hand side of Eq.(13) separately to zero. The underlying assumption being that the variations of Λ, G and c are such that conservation of energy, as true with the non varying parameters, is still valid. The solution for left hand side of Eq.(13), being zero, is

$$\rho_\Lambda = -\frac{\Lambda_0 s (c_0)^2}{8\pi G_0}\frac{a(2n + s - q)}{(2n + 3\gamma + s - q)} + Ba^{-3\gamma} \tag{14}$$

where B is a constant of integration. The right hand side of Eq.(13) can be written as,

$$\frac{\Lambda_0 (c_0)^2}{8\pi G_0}[\frac{sq}{2n + 3\gamma + s - q} - 2n]a^{2n+s-q-1} + \frac{3K(c_0)^2 n}{4\pi G_0}a^{2n-q-3} - qBa^{-3\gamma-1} = 0 \tag{15}$$

A solution for this equation is

$$\frac{sq}{2n + 3\gamma + s - q} = 2n, \tag{16}$$

$$2n - q - 3 = -3\gamma - 1, \tag{17}$$

and

$$\frac{3K(c_0)^2 n}{4\pi G_0} = qB. \tag{18}$$

Using Eq.(17), Eq.(16) can be written as,

$$\frac{qs}{s + 2} = 2n. \tag{19}$$

Then $q = \frac{3K(c_0)^2 n}{4\pi G_0 B} \equiv q_0 n$ and $s = \frac{4}{q_0 - 2}$.

For the dust era ($\gamma = 1$) Eq.(17) becomes

$$n = \frac{1}{q_0 - 2} \quad or \quad 4n = s \tag{20}$$

For n to be negative, as in the solution of the flatness problem, $q_0 < 2$. For the radiation dominated era ($\gamma = \frac{4}{3}$), Eq.(17) is

$$n = \frac{2}{q_0 - 2} \quad or \quad 2n = s \tag{21}$$

The Friedmann equation for a time varying cosmological parameter, Eq.(13), becomes,

$$\frac{\dot{a}^2}{a^2} = \frac{\Lambda_0 (c_0)^2}{3} \frac{(2n + 3\gamma - q)}{(2n + 3\gamma + s - q)} a^{2n+s} + \frac{8}{3}\pi B G_0 a^{q-3\gamma} - K(c_0)^2 a^{2n-2} \tag{22}$$

For the dust dominated universe, $\gamma = 1$, using Eq.(20)

$$\frac{\dot{a}^2}{a^2} = \frac{2\Lambda_0 (c_0)^2}{3(4n+2)} a^{6n} + \frac{8\pi B G_0}{3} a^{2n-2} - K(c_0)^2 a^{2n-2} \tag{23}$$

For $n < -\frac{1}{2}$ the cosmological term will go to zero at large times faster than the other two terms of Eq.(23). For $n < 1$ the curvature term will tend to zero at large times. Thus for $n < -\frac{1}{2}$ all the terms will vanish at large times solving both the cosmological and the flatness problems.

For the radiation dominated universe ($\gamma = \frac{4}{3}$), using equation (21)

$$\frac{\dot{a}^2}{a^2} = \frac{2\Lambda_0 (c_0)^2}{3(2n+2)} a^{4n} + \frac{8\pi B G_0}{3} a^{2n-2} - K(c_0)^2 a^{2n-2} \tag{24}$$

For $n < -1$ the Λ term will go to zero faster than the other two terms of the Eq.(24). As in the dust universe $n < 1$ makes the curvature term vanish at large times. Thus in the radiation dominated universe $n < -1$ solves both the cosmological and flatness problems.

We have solved the flatness and Λ problems in a Friedmann universe by assuming that the variation of c, G and Λ are such that the conservation of matter with fixed c, G and Λ still holds. In the absence of Λ the solution is the same as that without assuming energy conservation. With Λ we can still solve the cosmological problem but with different exponents. Thus it might be worth exploring a more fundamental theory that will allow the variation of the parameters c, G, Λ without violating energy conservation.

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